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Optimal product sizing through digital human models

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ABSTRACT

Designing for human variability (DfHV) requires efficient allocation of sizing and adjustability. This can preserve product performance while reducing some measures of cost. For example, specifying only as much adjustability as necessary for a desired level of accommodation leads to devices which are better suited to their users and more cost efficient. Similarly, when multiple sizes of an adjustable artifact are to be produced, specifying only as many sizes as are necessary, with an appropriate amount of adjustability per size, leads to a set of products that cost less, require fewer unique parts, facilitate maintenance standardization, and ease inventory control. An alternative to the standard procedure of evenly dividing size ranges is considered wherein an equal degree of accommodation per size is also presented. A simple example related to exercise bicycle seat height is discussed.

INTRODUCTION

In designing artifacts that interact with people with respect to size and physical configuration, the goal is to *accommodate* a certain percentage of users (Roe, 1993). Accommodation refers to the degree to which a design meets the needs of the user population, and it is often achieved through adjustability or the creation of separate sizes. The two approaches may be combined to produce an adjustable artifact available in several sizes. This work is mostly concerned with the design of such an artifact, composed of only one adjustable dimension. The general approach is one wherein specifying the amount of adjustability and the number and configuration of discrete sizes are in some ways separate considerations. These considerations are then superimposed for a final solution. Whether a solution is developed on paper, using physical models, or using digital human models, the topics presented in this paper are relevant for creating well-designed products.

While digital human modeling (DHM) is traditionally thought of as graphical, manikin-based design, the concept of representing individuals mathematically is much broader than that. It is a useful framework in which designs may be easily modified and evaluated with respect to their users. The current work considers digital human models as a context in which optimization methods may be used to determine a number of adjustable sizes that minimizes cost. These studies are conducted with thousands or virtual assessments instead of the traditional few.

SPECIFYING ADJUSTABILITY Efforts since the 1950's have produced tools that assist in basic assessment of accommodation. Often, the variability in body dimensions (called "anthropometry") is used to indicate how much *adjustability* is required to accommodate the intended user population. Adjustability is often specified in terms of required lower and upper limits to the adjustable product dimension under investigation. Many recommended tools and practices are in common use today that use anthropometry to prescribe adjustability (HFES 300 Committee, 2004; SAE International, 2006). Methods using only anthropometry include "manikin" approaches (Bittner, 2000; Diffrient et al., 1981; UGS, 2007) and population model approaches (Roe, 1993). So-called hybrid methods using attributes of both manikins and population models have been shown to be an improvement on using one or the other individually (Reed and Flannagan, 2000; Garneau and Parkinson, 2007).

Recent research has explored including a "preference" component, which considers variability not predicted by body dimensions (Reed and Flannagan, 2000; Parkinson et al., 2007; Parkinson and Reed, 2006a). In particular, Garneau and Parkinson (2007) outlines a method for determining the range of adjustability for a single-size, continuously adjustable artifact including preference effects.

Such a hybrid method including preference is based on the work of Parkinson et al. (2007) and Parkinson and Reed (2006b). Since this method will be used in this work, a brief overview of its usage is necessary.

The first step for implementing a hybrid method that incorporates preference is to obtain experimental data from a sample group using a prototype similar to the product being designed. These data must include some anthropometric measure from the sample group (often stature) as well as a corresponding preferred device configuration. Next, linear regression is performed to relate the user-selected setting to an anthropometric such as stature for the entire test population. Both the equation of the regression line and a measure of scatter (root-mean-square error, RMSE) must be retained. The parameters of regression are then used to construct a "virtual population" of a large size (e.g. 1000 members). Each member of this virtual sample is assigned a stature randomly drawn from a representative database (e.g. NHANES (Centers for Disease Control and Prevention, 2004) or ANSUR (Gordon et al., 1989)). A preferred setting for each member is determined using the results of the regression, including including a stochastic component calculated from the residual variance. The adjustability required to achieve the desired level of accommodation (e.g. 95%) is determined by taking the maximum and minimum selections of the appropriate portion (e.g., the central 95%) of virtual users.

SPECIFYING SIZES Surprisingly little research has been published regarding the specification of multiple sizes of an artifact. Most discrete sizing applications come from the apparel industry. McCulloch et al. (1998), which describes a method of using optimization to improve fit with multiple sizes of apparel, notes that most companies in the apparel industry do not have a systematic approach for specifying sizes, with largely anecdotal evidence guiding sizing choices. Specification of sizes is also often guided by meeting certain industry specifications or manufacturing tolerance. Tryfos (1985) and Tryfos (1986) confirm this observation and offer another application for optimization to minimize garment discomfort in order to maximize sales. HFES 300 Committee (2004) provides no general discussion of specifying sizes, but does provide one example of specifying sizes of military gloves in which the number of sizes is determined based on a minimum manufacturing tolerance.

A key difference between specifying sizes in the apparel industry and specifying sizes in general is that, for adjustable artifacts, fit is guaranteed for at least as many users as dictated by the adjustability solution (e.g. 95%). In other words, since the artifact is adjustable, there is no way to improve fit (or lessen disaccommodation) by specifying more sizes. The goal is then simply a matter of determining the optimum number and configuration (range) of sizes to minimize cost or some similar objective. Furthermore, nearly every application of size specification divides the dimensions of sizes evenly between the extremes, but in general, this may not be desirable.

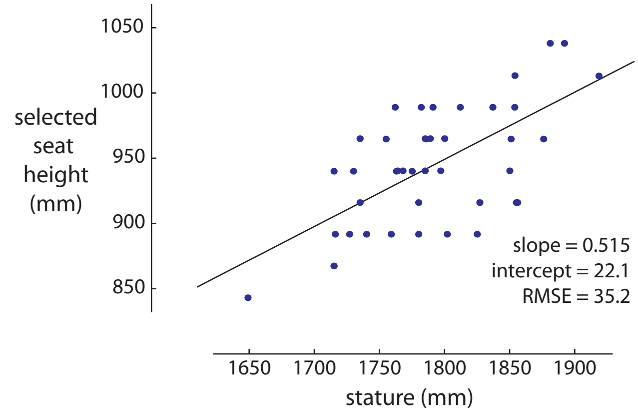

Figure 1: The selections of the 42-member sample used in Garneau and Parkinson (2007). The parameters of a linear regression and the resulting line are also shown.

The current work investigates the specification of multiple sizes of an adjustable artifact. A simple optimization problem is posed for determining the optimum number of sizes to minimize cost. Also, an alternative sizing scheme is explored, as opposed to the traditional approach of dividing sizes evenly.

METHODS

This analysis will use the same set of data as Garneau and Parkinson (2007); see that paper for a complete detailing of the experimental method and results—only a summary is provided here. The device to be designed is an upright exercise cycle and the target population is adult males. Therefore, the prototype is a typical upright exercise cycle (Pro-form XP70) and the sample group is a set of forty-two male engineering students at Penn State University. The metric of interest is the minimum seat height and its range of adjustability for an optimum number of sizes (with respect to cost). 95% accommodation of the target population is sought. It is important to note that this analysis is provided as a simple case study only, and is not intended to be a guide for bicycle seat height design. Figure 1 shows the selections of the 42 sample users and the associated linear regression.

SPECIFYING ADJUSTABILITY In order to determine the adjustable limits of each size, the limits of overall required adjustability must be determined. The method outlined in Garneau and Parkinson (2007) will be used because it has been shown to be more robust than traditional methods. Moreover, a virtual population as prescribed by this method will be required later in this analysis.

Linear regression is performed using the selected seat height and stature for the sample to create a *seat height* *preference model*. Then, the regression parameters of the seat height preference model are used to generate a virtual population of 1000 users randomly sampled from the ANSUR database. The preferred seat height, including a component indicating how their preference deviates from the mean, is calculated for each virtual user.

DETERMINING NUMBER OF SIZES In order to determine an optimum number of sizes with respect to cost, a cost function must be formulated. At its most basic, cost for an adjustable product with multiple sizes is proportional to the number of separate sizes, the quantity of each size produced, and the amount of adjustability within each size. Therefore, total cost may be broken into four components: fixed costs, the cost of offering a certain number of sizes, the cost of producing a given quantity of each size, and the cost of providing a certain amount of adjustability within each size. This may be represented mathematically as:

$$\mathrm{Cost}(n, q_i, \Delta X_i) = A + Bn + \sum_{i=1}^{n} \{C_i q_i + D q_i f(\Delta X_i)\} \quad (1)$$

where

$$n = \text{number of sizes}$$
$$q_i = \text{quantity of each size produced}$$
$$\Delta X_i = \text{amount of adjustability in each size}$$
$$f(\Delta X_i) = \text{function relating adjustability to cost}$$

The following constraints must also be satisfied:

$$\sum_{i=1}^{n} q_i = q \quad (2)$$

$$\sum_{i=1}^{n} \Delta X_i = \Delta X \quad (3)$$

where

$$q = \text{total quantity of products produced}$$
$$\Delta X = \text{total overall adjustable range}$$

The constants in Equation 1 are defined as follows: A represents fixed costs such as marketing and capital expenses. B is the cost of offering a size. This might include the cost of storing a size and reconfiguring machinery to produce different sizes, for example. C_i is the manufacturing cost per product for each size. In general, C_i may be different for each size and may be a function of the quantity of that size produced. D is the incremental cost of adjustability in each size. Each constant must be determined on a case-by-case basis.

Even sizing over adjustment range When the sizes are evenly distributed over the adjustable range, the required adjustability per size, ΔX_i, is the same for all sizes and is given by $\Delta X/n$. Figure 2 demonstrates the inverse nature of this relationship for the exercise cycle in which $\Delta X = 192$ mm. An even distribution of sizes simplifies Equation 1, since $\Delta X/n$ may be substituted for ΔX_i.

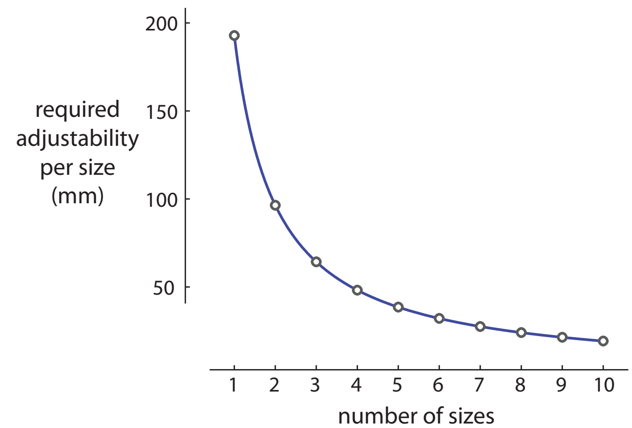

Figure 2: Required adjustability per size versus number of sizes for the exercise cycle example with each size having equal adjustability.

Two more assumptions make a simpler form of Equation 1 possible. First, C_i is assumed to be the same across all sizes and independent of the quantity of each size produced. Combined with the constraint of Equation 2, this allows the cost of production to simply become Cq when summed across all sizes. Second, a simple function is assumed for $f(\Delta X_i)$. In general, any function appropriate to the application may be used. Here a quadratic form will be assumed such that $f(\Delta X_i)$ becomes $(\Delta X/n)^2$. This models the nonlinear increase in cost with increase in adjustability. Note that in the case of evenly distributed sizes, although ΔX_i is the same for all sizes, q_i is not.

The cost objective function is now only a function of the number of sizes, if a certain total quantity q and amount of overall adjustability ΔX are assumed. Formatting this objective function in the standard way gives:

$$\min\ \mathrm{Cost}(n) = A + Bn + Cq + Dq\left(\frac{\Delta X}{n}\right)^2 \quad (4)$$

Neglecting $A + Cq$ (since it is a constant for any number of sizes), a relative cost function may be formatted as:

$$\min\ \mathrm{Cost}(n) = \frac{B}{D}n + q\left(\frac{\Delta X}{n}\right)^2 \quad (5)$$

From this equation, one can see that an increase in the number of sizes increases production costs but decreases the cost of adjustability (since each size requires less adjustability). Therefore, B/D may be thought of as a penalty for adding additional sizes, or as the degree to which an increase in the number of sizes increases cost over an increase in the amount of adjustability per size.

Uneven sizing over adjustment range   Most existing sizing schemes divide the range prescribed by the size extrema evenly among equal sizes. Although such a distribution yields sizes of equal range, an equal number of users per size is not achieved. This can lead to unbalanced inventories and increased cost of production for the sizes for which there is less demand. Therefore, an alternate sizing scheme may be employed that attempts to accommodate an equal number of users per size. This also means that an equal quantity of each size is to be produced. Thus, q_i is the same for all sizes and is equal to q/n. This requires that the adjustability for each size, ΔX_i, is different across sizes. Assuming C_i is a constant across sizes as in the last case, the cost objective function then becomes:

$$\min \text{Cost}(n, \Delta X_i) = A + Bn + Cq + D\left(\frac{q}{n}\right)\sum_{i=1}^{n} f(\Delta X_i) \quad (6)$$

$$\text{subject to } \sum_{i=1}^{n} \Delta X_i = \Delta X \quad (7)$$

Equation 6 is easily simplified only if $f(\Delta X_i)$ is a linear function, i.e. the cost of adjustability is linearly related to cost. Therefore, a simplified cost objective function similar to Equation 4 is not possible for a quadratic $f(\Delta X_i)$. If $f(\Delta X_i)$ is linear, then optimizing cost with respect to sizes for evenly or unevenly distributed sizes yields the same results.

How may the adjustment ranges for equal accommodation per size be determined? For a simple univariate linear problem, such as the exercise cycle example considered later in this paper, traditional methods such as boundary manikins may be used to prescribe adjustment range cutoffs for equal accommodation per size (hence equal quantities per size). This is done by determining the appropriate-percentile values for the dimension under consideration. For example, if there are to be three sizes with 95% accommodation, the designer would look to the 2.5th, 34th, 66th, and 97.5th percentile values of the dimension under consideration to determine the limits of the adjustable range. In more complex multivariate problems, however, traditional methods for determining adjustable limits do not provide a mechanism to determine a sizing scheme in which an equal number of persons are accommodated per size. However, using the virtual population method described earlier, determining equal-accommodation size limits is achievable even for multivariate problems. A number of users equal to $(p-a)/n$ may be attributed to each size, where p is the virtual population size (e.g. 1000) and a is the degree of accommodation (e.g. $a = 50$ for 95% accommodation with $p = 1000$). This method is used in the next section for the exercise cycle example.

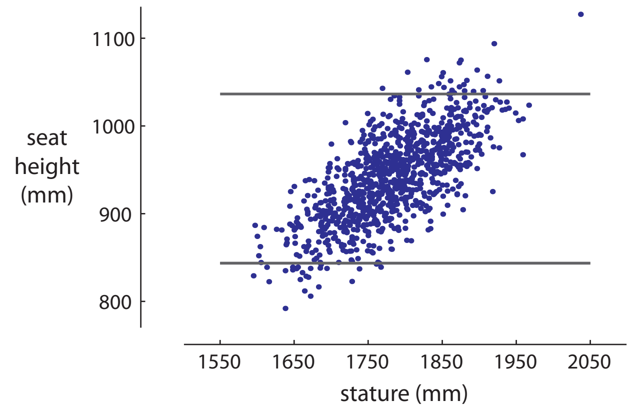

Figure 3: The stature and seat selections of the 1000 member virtual sample are plotted (includes preference). Adjustment limits for 95% accommodation are denoted by the horizontal lines.

RESULTS

The virtual 1000-member sample generated from the regression parameters given by Figure 1 are shown in Figure 3. It also shows the overall design limits to accommodate 95% of the target population. It is the goal of the various sizing schemes to determine how to divide this overall range.

Figure 4 shows cost curves in the case where the n sizes are divided evenly, $q = 1000$, and $\Delta X = 0.192$ m. Fixed costs are neglected and so the curves represent relative costs for various values of B/D. The optimum number of sizes for each curve is given by the minimum cost. Suppose the value of B/D for the exercise cycle is determined to be about 2.5. Then the optimum number of sizes to minimize cost is about 3, determined by inspecting Figure 4 or by using an unconstrained optimization algorithm. Notice that an increasing value of B/D means that offering more sizes becomes increasingly expensive over adding more adjustability per size. Therefore, the optimum number of sizes decreases, as shown in the plot.

If the size limits of the exercise cycle are evenly distributed as shown in Figure 5, then about 47% are targeted by the middle band, and roughly 24% are targeted by the outer bands. Therefore, $\Delta X_{1,2,3} = 64$ mm, $q_1 = q_3 = 0.24q$ and $q_2 = 0.47q$. On the other hand, if the size limits are as prescribed by the nonlinear scheme of Figure 6 , then the proportion of the target population accommodated by each size will be evenly distributed. Therefore, $q_{1,2,3} = 0.32q$, $\Delta X_1 = 77$ mm, $\Delta X_2 = 42$ mm, and $\Delta X_3 = 74$ mm. These nonlinear size limits are determined simply by determining the limits of sizes for which $950/n$ users in the virtual population are attributed to each size, where $n = 3$.

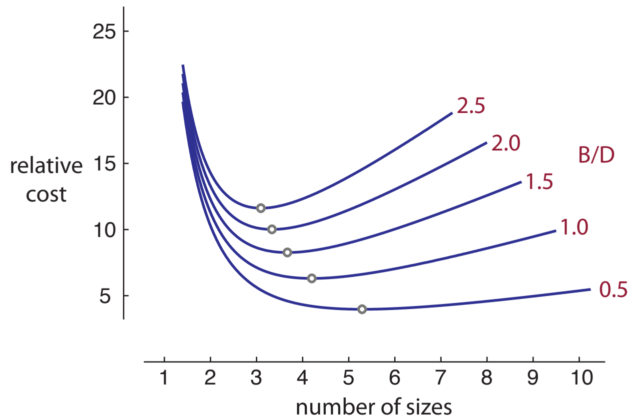

Figure 4: Relative cost as a function of the number of sizes and the constant B/D for the exercise cycle example with sizes of equal adjustability. Various values for this constant are given to the right of the curves. The point of minimum cost is also shown on each curve.

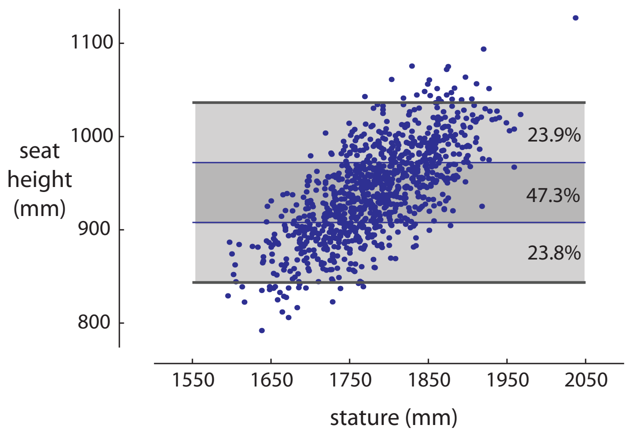

Figure 5: The size limits for an evenly spaced sizing scheme are shown along with the 1000 member virtual sample and the accommodation for each size. Note the varying levels of accommodation for each size.

DISCUSSION

Consider the case of the exercise cycle with evenly distributed sizes. Figure 2 indicates a convergence of required adjustability as n increases, along with diminishing marginal returns beyond an n of, say, 4 or 5. Figure 4 similarly shows that cost is minimized at between 3 and 5 sizes for most reasonable values of B/D. This supports common knowledge in design that an appropriate number of sizes most often is between 2 and 6.

Certain assumptions drastically simplify the solution for determining an optimum number of sizes based on cost. Making the assumption that C_i is constant across all sizes means that cost will always be higher for unevenly distributed sizes if $f(\Delta X_i)$ is an appropriate nonlinear relationship (e.g. quadratic). So if the cost of production, C_i, is constant across all sizes, the cheapest way to manufacture an adjustable product with multiple sizes is to divide the range of adjustability equally over the sizes. The

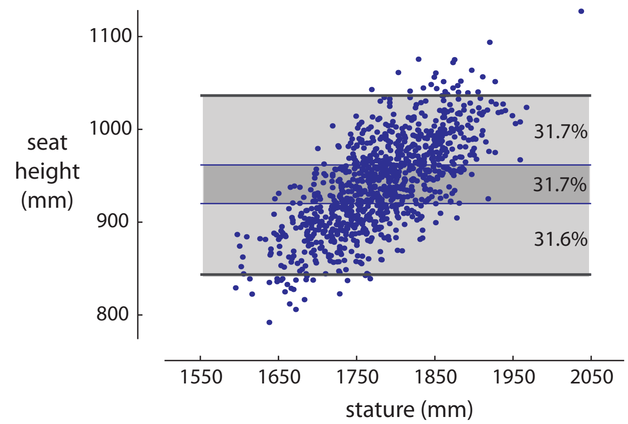

Figure 6: The size limits for a uniform accommodation sizing scheme are shown along with the 1000 member virtual sample and the accommodation for each size. Note the equal levels of accommodation but different adjustable range per size.

methodology, however, is not dependent upon these assumptions.

Why might it be desirable to unevenly divide the adjustment range across the sizes? That is, why might targeting an equal number of users per size be desirable? In practice, C_i is not constant across sizes and is a function of the quantity sold per size, q_i. If different quantities of each size are produced, it will cost more to manufacture lesser quantities of certain sizes. Thus, total cost may be less if equal quantities of each size are produced, depending on the nature of C_i and $f(\Delta X)$. This scenario will be further investigated in future research. Similarly, future research will also more rigorously investigate the nonlinear sizing scheme introduced in this paper by applying formal optimization algorithms for minimizing cost in the nonlinear scheme.

CONCLUSION

The results of this paper show that a systematic approach toward determining the number of sizes for minimum cost is viable, but requires some judgment and assumptions in determining appropriate constants and relationships to develop a cost function. Existing research shows that the number of sizes may be determined by manufacturing tolerance or anecdotal evidence, but such a scheme may not lead to optimal results. Targeting an even number of users per size and thus producing equal quantities of all sizes might make production and supply of multiple sizes more efficient. Achieving an equal number of users per size is not feasible with many traditional methods of determining adjustability, and the ability of the virtual population method to do so is a distinct advantage of the method.

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